AQA

A-level
FURTHER MATHEMATICS
7367/2

Paper 2

Mark scheme

June 2024

Version: 1.0 Final

246A7367/2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles 3rd answer	1.1b	B1	8
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles 2nd answer	2.2a	B1	45 m s^{-1}
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks 3rd box	2.2a	B1	$0 < g(x) \leq 1$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Circles 4th answer	2.2a	B1	$(x^2 + 25)$
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains $\sum_{r=1}^{n} r(r+1)$ OE ISW Condone $\sum_{r=1}^{n} r^2 + r$	2.5	B1	$\sum_{r=1}^{n} r(r+1)$
	Subtotal		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	Uses $\frac{1}{6}n(n+1)(2n+1)$ and $\frac{1}{2}n(n+1)$	1.1a	M1	$\sum_{r=1}^{n}(r^2+r) = \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$ $= \frac{1}{6}n(n+1)(2n+1+3)$ $= \frac{1}{3}n(n+1)(n+2)$
	Completes fully correct working, with at least one intermediate step, to obtain $\frac{1}{3}n(n+1)(n+2)$ AG LHS of typical solution not required.	2.1	R1	
	Subtotal		**2**	

	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
6	Sets $y = 3x$ PI by correct substitution or Obtains one of $\sum 3\alpha = \pm 15$ $\sum (3\alpha)(3\beta) = \pm 36$ $(3\alpha)(3\beta)(3\gamma) = \pm 54$	1.1a	M1	Let $y = 3x$ Then $x = \frac{y}{3}$ $\frac{y^3}{27} + \frac{5y^2}{9} - \frac{4y}{3} + 2 = 0$ $y^3 + 15y^2 - 36y + 54 = 0$
	Replaces x with $\frac{y}{3}$ or $3y$ Accept x for y or Obtains at least two of $\sum 3\alpha = \pm 15$ $\sum (3\alpha)(3\beta) = \pm 36$ $(3\alpha)(3\beta)(3\gamma) = \pm 54$	1.1a	M1	
	Obtains $y^3 + 15y^2 - 36y + 54 = 0$ OE with integer coefficients.	1.1b	A1	
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7	Calculates **AB** or **BA** with at least three correct elements.	1.1a	M1	**AB** $= \begin{bmatrix} p-2 & p-1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2p-1 \\ 0 & 4-p \end{bmatrix}$ $= \begin{bmatrix} p-2 & (p-2)(2p-1)+(p-1)(4-p) \\ 0 & 4-p \end{bmatrix}$ $= \begin{bmatrix} p-2 & p^2-2 \\ 0 & 4-p \end{bmatrix}$
	Calculates **AB** and **BA** with at least seven correct elements.	1.2	M1	
	Uses **AB** = **BA** to form a quadratic equation in p and obtains at least one solution	1.1a	M1	**BA** $= \begin{bmatrix} 1 & 2p-1 \\ 0 & 4-p \end{bmatrix}\begin{bmatrix} p-2 & p-1 \\ 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} p-2 & p-1+2p-1 \\ 0 & 4-p \end{bmatrix}$ $= \begin{bmatrix} p-2 & 3p-2 \\ 0 & 4-p \end{bmatrix}$
	Completes a reasoned argument to obtain $p = 0, 3$ Must have stated that since **A** and **B** are commutative (under matrix multiplication,) **AB** = **BA**	2.1	R1	**A** and **B** are commutative, hence **AB** = **BA** Thus $p^2 - 2 = 3p - 2$ $p(p - 3) = 0$ $p = 0, 3$
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8	Uses distributive property of cross-product. Condone one numerical error.	3.1a	M1	$(\mathbf{a} - 4\mathbf{b} + 3\mathbf{c}) \times (2\mathbf{a}) = \mathbf{a} \times 2\mathbf{a} - 4\mathbf{b} \times 2\mathbf{a} + 3\mathbf{c} \times 2\mathbf{a}$ $= \mathbf{0} + 8\mathbf{a} \times \mathbf{b} - 6\mathbf{a} \times \mathbf{c}$ $= \begin{bmatrix} 16 \\ 8 \\ -18 \end{bmatrix}$
	Uses $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ PI by only $\mathbf{b} \times \mathbf{a}$ and $\mathbf{c} \times \mathbf{a}$ terms remaining	1.1a	M1	
	Uses anti-commutative property of cross-product at least once.	1.1a	M1	
	Obtains $\begin{bmatrix} 16 \\ 8 \\ -18 \end{bmatrix}$	1.1b	A1	
	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
9	Uses Euler's method once to calculate an estimate of y when $x = -1.98$ Condone one error.	1.1a	M1	$x_0 = -2$ $y_0 = 4.73$ $h = 0.02$ $y_1 = 4.73 + 0.02\left(\dfrac{4.73^2 - (-2)^2}{2(-2) + 3(4.73)}\right)$ $= 4.766060648$ $x_1 = -1.98$ $y_2 = 4.73 + 0.04\left(\dfrac{4.766060648^2 - (-1.98)^2}{2(-1.98) + 3(4.766060648)}\right)$ $= 4.8027$
	Obtains AWRT 4.766 PI by final AWRT 4.8027	1.1b	A1	
	Uses midpoint formula once with their y_1 Condone one error.	3.1a	M1	
	Obtains AWRT 4.8027	1.1b	A1	
	Question total		**4**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td>10</td><td>Obtains $3x+2kx$ and $-4x+5kx$
Accept any letter for k

Condone use of $y=kx+c$

Or
Uses det (C – λI)</td><td>1.1a</td><td>M1</td><td rowspan="4">For an invariant line $y = kx$
$\begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} x \\ kx \end{bmatrix}=\begin{bmatrix} x' \\ y' \end{bmatrix}$
$3x+2kx=x'$
$-4x+5kx=y'$
$y'=kx'$
$-4x+5kx=k(3x+2kx)$
$-4+5k=2k^2+3k$
$2k^2-2k+4=0$
$\Delta=2^2-32=-28$
$\Delta<0$

The equation has no real roots, so there is no invariant line of the form $y = kx$.</td></tr>
<tr><td></td><td>Substitutes their x' and y' in $y'=kx'$
Accept any letter for k
Condone use of $y=kx+c$

Or
Expands det (C – λI)</td><td>1.1a</td><td>M1</td></tr>
<tr><td></td><td>Deduces
$2k^2-2k+4=0$ OE

Or

$\lambda^2-8\lambda+23=0$</td><td>2.2a</td><td>A1</td></tr>
<tr><td></td><td>Completes a reasoned argument justifying that the quadratic equation has no real roots to prove that the transformation represented by C has no invariant lines of the form $y = kx$
If $y=kx+c$ is used then must state and use $c = 0$</td><td>2.1</td><td>R1</td></tr>
<tr><td></td><td>Question total</td><td></td><td>4</td><td></td></tr>
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Q	Marking instructions	AO	Marks	Typical solution
11	Refers to polynomials of odd and/or even degree. Or States that a polynomial of a particular odd degree greater than 3 can have exactly one real root or sketches a graph to show this. or Obtains a polynomial of degree greater than three with exactly one real root.	2.4	M1	The polynomial equation $z^5 - 1 = 0$ is of degree 5 and has exactly one real root. This is a counter example to Latifa's statement. So Sam is right.
	Explains that complex roots occur in conjugate pairs (condone "imaginary roots"). or Explains that their specific polynomial is a counter example to Latifa's statement.	2.4	M1	
	Completes a reasoned argument to conclude that Sam is right (do not condone "imaginary roots") and States clearly that Sam is right. OE	2.3	R1	
	Question total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
12	Obtains $\begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$	2.2a	B1	T is a reflection in the line $y = x\tan\frac{\pi}{3}$ So $\mathbf{N} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ $\mathbf{M}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -6 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-6y \\ 2x+7y \end{bmatrix}$ $\begin{bmatrix} 3 \\ 8 \end{bmatrix} = \mathbf{N}\left(\mathbf{M}\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -\frac{1}{2}(x-6y)+\frac{\sqrt{3}}{2}(2x+7y) \\ \frac{\sqrt{3}}{2}(x-6y)+\frac{1}{2}(2x+7y) \end{bmatrix}$ $\begin{bmatrix} 3 \\ 8 \end{bmatrix} = \begin{bmatrix} \left(\sqrt{3}-\frac{1}{2}\right)x+\left(3+\frac{7\sqrt{3}}{2}\right)y \\ \left(\frac{\sqrt{3}}{2}+1\right)x+\left(\frac{7}{2}-3\sqrt{3}\right)y \end{bmatrix}$ $1.23205x+9.06218y=3$ $1.86603x-1.69615y=8$ $x=4.083,\ y=-0.224$ $P(4.083, -0.224)$ (3 d.p.)
	Operates first **M**, then their **N**, on column vector $\begin{bmatrix} x \\ y \end{bmatrix}$ or Operates their **N** or their $\mathbf{N}^{-1}$ on $\begin{bmatrix} 3 \\ 8 \end{bmatrix}$ or Calculates matrix product **NM**	3.1a	M1	
	Obtains a correct system of simultaneous equations in x and y or Obtains correct value of $\mathbf{M}^{-1}$ (Accept decimal approximation.) or Obtains correct value of **NM** PI	1.1b	A1	
	Solves their simultaneous equations in x and y from $\mathbf{NM}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ or $\mathbf{MN}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$ or Operates $\mathbf{M}^{-1}$ on **NQ** or Uses inverse of **NM**, where $(\mathbf{NM})^{-1} = \begin{bmatrix} 0.08927 & 0.47696 \\ 0.09821 & -0.06484 \end{bmatrix}$ to obtain coordinates of P Condone applying transformations in the wrong order.	1.1a	M1	

	Obtains AWRT 4.083 and AWRT -0.224. Condone column vector form if x and y seen. Accept exact values. $x = \frac{27+74\sqrt{3}}{38}$, $y = \frac{14-13\sqrt{3}}{38}$	1.1b	A1	
	Question total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Uses partial fractions.	3.1a	M1	$\frac{1}{(r-1)r(r+1)} \equiv \frac{A}{r-1}+\frac{B}{r}+\frac{C}{r+1}$ $1 \equiv Ar(r+1)+B(r-1)(r+1)+C(r-1)r$ $r=0: \quad 1=-B \Rightarrow B=-1$ $r=1: \quad 1=2A \Rightarrow A=\frac{1}{2}$ $r=-1: \; 1=2C \Rightarrow C=\frac{1}{2}$
	Obtains $\frac{1}{2(r-1)}-\frac{1}{r}+\frac{1}{2(r+1)}$ OE	1.1b	A1	
	Writes at least 3 consecutive rows of the sum	1.1a	M1	$\sum_{r=2}^{n}\frac{1}{(r-1)r(r+1)}=\sum_{r=2}^{n}\left(\frac{1}{2(r-1)}-\frac{1}{r}+\frac{1}{2(r+1)}\right)$ $=\frac{1}{2(1)}-\frac{1}{2}+\frac{1}{2(3)}$ $+\frac{1}{2(2)}-\frac{1}{3}+\frac{1}{2(4)}$ $+\frac{1}{2(3)}-\frac{1}{4}+\frac{1}{2(5)}$ $+\ldots\ldots$ $+\frac{1}{2(n-3)}-\frac{1}{n-2}+\frac{1}{2(n-1)}$ $+\frac{1}{2(n-2)}-\frac{1}{n-1}+\frac{1}{2n}$ $+\frac{1}{2(n-1)}-\frac{1}{n}+\frac{1}{2(n+1)}$
	Uses the method of differences showing at least the first three and last two terms (or vice versa)	2.5	M1	
	Completes fully correct working to reach the required result. AG	2.1	R1	$\sum_{r=2}^{n}\frac{1}{(r-1)r(r+1)}=\frac{1}{2}-\frac{1}{2}+\frac{1}{4}+\frac{1}{2n}-\frac{1}{n}+\frac{1}{2(n+1)}$ $=\frac{1}{4}-\frac{1}{2n}+\frac{1}{2(n+1)}$
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Obtains an inequality in *n* PI	3.1a	M1	$\sum_{r=2}^{n}\frac{1}{(r-1)r(r+1)} > 0.24999$ $\frac{1}{4}-\frac{1}{2n}+\frac{1}{2(n+1)} > 0.24999$
	Rearranges their inequality or equation to obtain a quadratic inequality or equation. PI	1.1a	M1	$0.00001 > \frac{1}{2n}-\frac{1}{2(n+1)}$ $0.00001 > \frac{1}{2n(n+1)}$
	Deduces the correct value of *n*	2.2a	A1	$2n(n + 1) > 10^5$ $2n^2 + 2n - 10^5 > 0$ Solutions to $2n^2 + 2n - 10^5 = 0$ are 223.1 and –224.1 $n > 223.1$ $n = 224$
	Subtotal		**3**	

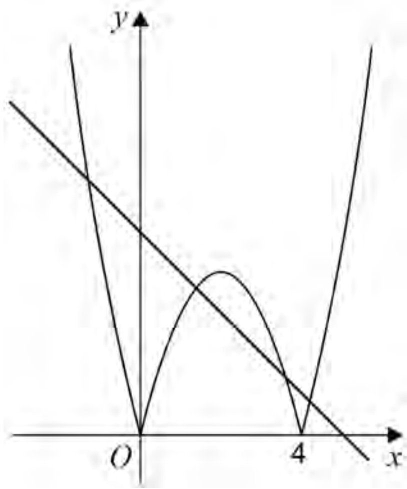
	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Obtains $12 - 2k$	1.1b	B1	$\lvert\mathbf{M}\rvert = 5(15 - 2k - 3) - 2(30 - 4k - 6) + (6 - 6)$ $= 12 - 2k$ Cofactors are $\begin{bmatrix} -2k+12 & 4k-24 & 0 \\ -9 & 23 & -1 \\ 4k+3 & -10k-9 & 3 \end{bmatrix}$ $\mathbf{M}^{-1} = \dfrac{1}{12-2k}\begin{bmatrix} -2k+12 & -9 & 4k+3 \\ 4k-24 & 23 & -10k-9 \\ 0 & -1 & 3 \end{bmatrix}$
	Obtains matrix of minors/cofactors with at least **four** correct elements. PI by transposed form. Condone overall sign error on each element.	1.1a	M1	
	Obtains matrix of minors/cofactors with at least **seven** correct elements. PI transposed form. Condone overall sign error on each element.	1.1a	M1	
	Obtains correct matrix of minors/cofactors. PI transposed form. Condone overall sign error on one element.	1.1a	M1	
	Obtains fully correct, simplified answer.	1.1b	A1	
	Subtotal		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(b)	Obtains $k \neq 6$ Follow through their determinant.	1.1b	B1F	$k \neq 6$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
14(c)	Uses their $\mathbf{M}^{-1}$ to form a product to find the solution set. Must include $\begin{bmatrix}1\\4k+3\\9\end{bmatrix}$ or Obtains $\mathbf{M}^{-1}\begin{bmatrix}1\\4k+3\\9\end{bmatrix}$ for a particular value of k	3.1a	M1	$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{12-2k}\begin{bmatrix}-2k+12 & -9 & 4k+3\\4k-24 & 23 & -10k-9\\0 & -1 & 3\end{bmatrix}\begin{bmatrix}1\\4k+3\\9\end{bmatrix}$ $=\frac{1}{12-2k}\begin{bmatrix}-2k+12-36k-27+36k+27\\4k-24+92k+69-90k-81\\-4k-3+27\end{bmatrix}$ $=\frac{1}{12-2k}\begin{bmatrix}-2k+12\\-36+6k\\24-4k\end{bmatrix}$ $=\begin{bmatrix}1\\-3\\2\end{bmatrix}$ which is independent of k
	Obtains at least one correct component from their $\mathbf{M}^{-1}$, can be unsimplified. or Obtains correct $\mathbf{M}^{-1}\begin{bmatrix}1\\4k+3\\9\end{bmatrix}$ for their k	1.1b	A1F	
	Obtains at least two correct components from their $\mathbf{M}^{-1}$ (can be unsimplified). or Correctly substitutes their $\mathbf{M}^{-1}\begin{bmatrix}1\\4k+3\\9\end{bmatrix}$ Into equation.	1.1b	A1F	
	Uses correct reasoning to obtain the required result.	2.1	R1	
	Subtotal		**4**	

	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Draws curve with basically correct shape and no negative y-values.	1.1b	B1	
	Their graph intersects line at four distinct points.	1.1b	B1	
	Value of 4 shown at x-intercept	1.1b	B1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Uses the modulus function to obtain two separate quadratic equations.	3.1a	M1	$x^2 - 4x = 5 - x$ $x^2 - 3x - 5 = 0$ $x = \dfrac{3 \pm \sqrt{29}}{2}$ $4x - x^2 = 5 - x$ $0 = x^2 - 5x + 5$ $x = \dfrac{5 \pm \sqrt{5}}{2}$ $x < \dfrac{3-\sqrt{29}}{2}, \dfrac{5-\sqrt{5}}{2} < x < \dfrac{5+\sqrt{5}}{2}, x > \dfrac{3+\sqrt{29}}{2}$
	Obtains four correct x-values of points of intersection (condone decimal approximations).	1.1a	A1	
	Uses their graph to obtain at least one subset of the solution set (condone decimal approximations).	2.2a	M1	
	Deduces a completely correct solution set with exact values.	2.2a	A1	
	Subtotal		**4**	

	Question total		**7**	

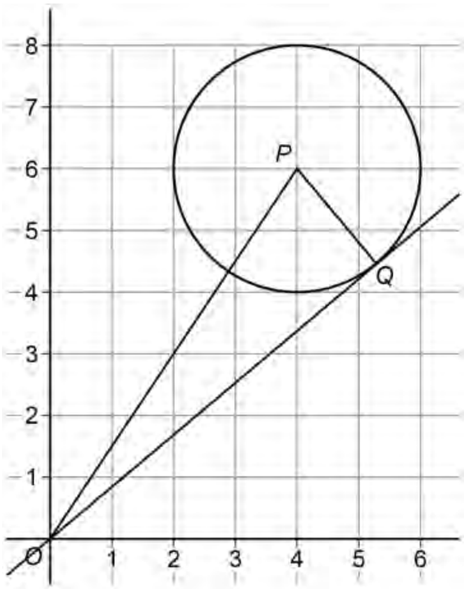
Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Deduces that $a = 3$	2.2a	B1	$a = 3$, $b = 2$
	Deduces that $b = 2$	2.2a	B1	
	Subtotal		**2**	

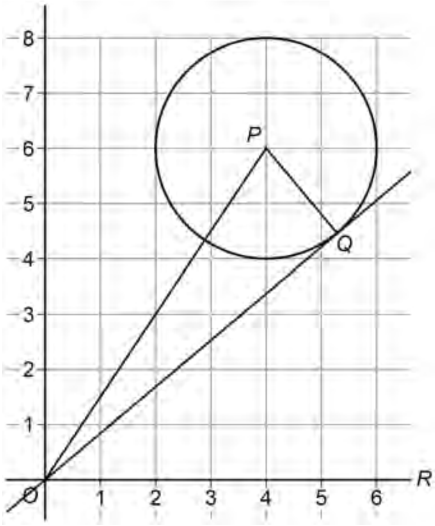
Q	Marking instructions	AO	Marks	Typical solution
16(b)(i)	Deduces that x-intercept is $-\frac{5}{3}$. PI by correct answer.	2.2a	B1	$\text{f}(x) = \dfrac{3x+5}{x+2}$ $x\text{-intercept} = -\dfrac{5}{3}$ $V = \pi\displaystyle\int_{-\frac{5}{3}}^{0}\left(\dfrac{3x+5}{x+2}\right)^2 \text{d}x$ = 21.2040… = 21.2
	Uses $\pi\int y^2 \text{d}x$ Condone missing dx and missing/incorrect limits. PI by correct answer.	1.1a	M1	
	Obtains AWRT 21.2	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(ii)	Deduces that y-intercept = 2.5 PI correct answer.	2.2a	B1	$y\text{-intercept} = 2.5$ $y = \dfrac{3x+5}{x+2}$ $yx + 2y = 3x + 5$ $x(y-3) = 5-2y$ $x = \dfrac{5-2y}{y-3}$ $V = \pi\displaystyle\int_{0}^{2.5}\left(\dfrac{5-2y}{y-3}\right)^2 \text{d}y$ = 14.1360… = 14.1
	Deduces an expression for x in terms of y PI correct answer.	2.2a	M1	
	Uses $\pi\int x^2 \text{d}y$ Condone missing dy, use of dx and missing/incorrect limits. PI by correct answer.	1.1a	M1	
	Obtains AWRT 14.1	1.1b	A1	
	Subtotal		**4**	

	Question total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Obtains $4+6\text{i}$ Allow $\lvert z-4-6\text{i}\rvert$	1.2	B1	$\lvert z-(4+6\text{i})\rvert = 2$
	Obtains $a = 2$	1.2	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)(i)	Correctly identifies the point representing z_1 PI by correct method. or Obtains $m = \frac{6-2\sqrt{3}}{3}$ as the gradient of the tangent at z_1	2.2a	B1	 Q is the point representing z_1 $OP^2 = 52$ $PQ^2 = 4$ $OQ^2 = 48$ $OQ = 4\sqrt{3}$ $\lvert z_1\rvert = 4\sqrt{3}$
	Uses Pythagoras or other correct method to obtain $\lvert z_1\rvert$	3.1a	M1	
	Obtains $4\sqrt{3}$ Accept any exact correct value eg $\sqrt{48}$	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
17(b)(ii)	Deduces that $\arg z_1 = \widehat{POR} - \widehat{POQ}$ or Uses $\tan(\arg(z_1)) = \dfrac{6-2\sqrt{3}}{3}$ or Obtains $x = \dfrac{48+12\sqrt{3}}{13}$ or $y = \dfrac{72-8\sqrt{3}}{13}$ where $z_1 = x + \mathrm{i}y$	2.2a	B1	 $\sin \widehat{POQ} = \dfrac{2}{\sqrt{52}} = \dfrac{1}{\sqrt{13}}$ $\cos \widehat{POQ} = \dfrac{4\sqrt{3}}{\sqrt{52}} = \dfrac{2\sqrt{3}}{\sqrt{13}}$ $\sin \widehat{POR} = \dfrac{6}{\sqrt{52}} = \dfrac{3}{\sqrt{13}}$ $\cos \widehat{POR} = \dfrac{4}{\sqrt{52}} = \dfrac{2}{\sqrt{13}}$ $\arg z_1 = \widehat{POR} - \widehat{POQ}$ $\sin(\arg(z_1)) = \dfrac{3}{\sqrt{13}} \times \dfrac{2\sqrt{3}}{\sqrt{13}} - \dfrac{2}{\sqrt{13}} \times \dfrac{1}{\sqrt{13}} = \dfrac{6\sqrt{3}-2}{13}$ $\arg(z_1)$ is acute, so $\arg(z_1) = \arcsin\left(\dfrac{6\sqrt{3}-2}{13}\right)$
	Uses a suitable trigonometric identity or Uses a correct method to obtain $\sin(\arg(z_1))$ from $\tan(\arg(z_1))$ or Obtains $x = \dfrac{48+12\sqrt{3}}{13}$ and $y = \dfrac{72-8\sqrt{3}}{13}$ where $z_1 = x + \mathrm{i}y$	1.1a	M1	
	Obtains sines and cosines of $\widehat{POR}$ and $\widehat{POQ}$ (at least three correct) or Obtains $\sin^2(\arg(z_1))$ or $\cos^2(\arg(z_1))$ or Obtains the values of the sides of a right-angled triangle with an angle equal to $\arg(z_1)$	3.1a	M1	
	Uses correct reasoning to obtain the required result. Condone omission of "$\arg(z_1)$ is acute". AG	2.1	R1	
	Subtotal		**4**	

	Question total		**9**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="4">18</td><td>Substitutes $\sin\theta$ for x in the first four terms of the correct binomial series with $n = -4$.
Condone sign errors.</td><td>1.1a</td><td>M1</td><td rowspan="4">$(1+\sin\theta)^{-4} = 1 + (-4)\sin\theta + \frac{(-4)(-5)}{2}\sin^2\theta + \frac{(-4)(-5)(-6)}{6}\sin^3\theta + \ldots$
$= 1 - 4\left(\theta - \frac{\theta^3}{6} + \ldots\right) + 10\left(\theta - \frac{\theta^3}{6} + \ldots\right)^2$
$-20\left(\theta - \frac{\theta^3}{6} + \ldots\right)^3 + \ldots$
$= 1 - 4\theta + \frac{2\theta^3}{3} + 10\theta^2 - 20\theta^3 + o\left(\theta^4\right)$
$\frac{1}{(1+\sin\theta)^4} = 1 - 4\theta + 10\theta^2 - \frac{58}{3}\theta^3 + \ldots$</td></tr>
<tr><td>Substitutes in their binomial series the first two terms of the sine series in the first bracket, and the first term of the sine series in subsequent brackets.</td><td>3.1a</td><td>M1</td></tr>
<tr><td>Obtains the correct series.
ISW
Condone x used for θ</td><td>1.1b</td><td>A1</td></tr>
<tr><td>Discards higher powers of θ in a correct, fully simplified, series</td><td>2.2a</td><td>A1</td></tr>
<tr><td></td><td>Question total</td><td></td><td>4</td><td></td></tr>
</table>

Q	Marking instructions	AO	Marks	Typical solution
19	Uses a three-term auxiliary equation to obtain the Complementary Function.	3.1a	M1	Complementary Function $\lambda^2 + 4\lambda - 45 = 0$ $\lambda = 5$ or $\lambda = -9$ $y = Ae^{-9x} + Be^{5x}$ Particular Integral $y_{PI} = Cxe^{5x} + D + Ex + Fx^2$ $y'_{PI} = Ce^{5x} + 5Cxe^{5x} + E + 2Fx$ $y''_{PI} = 10Ce^{5x} + 25Cxe^{5x} + 2F$ $10Ce^{5x} + 25Cxe^{5x} + 2F + 4(Ce^{5x} + 5Cxe^{5x} + E + 2Fx)$ $-45(Cxe^{5x} + D + Ex + Fx^2) = 21e^{5x} - 0.3x + 27x^2$ $\Rightarrow C = \frac{3}{2},\ D = \frac{-8}{225},\ E = \frac{-1}{10},\ F = \frac{-3}{5}$ General Solution $y_{GS} = Ae^{-9x} + Be^{5x} + \frac{3}{2}xe^{5x} - \frac{8}{225} - \frac{1}{10}x - \frac{3}{5}x^2$ $y'_{GS} = -9Ae^{-9x} + 5Be^{5x} + \frac{3}{2}e^{5x} + \frac{15}{2}xe^{5x} - \frac{1}{10} - \frac{6}{5}x$ $x = 0$ $\frac{37}{225} = A + B - \frac{8}{225} \Rightarrow A + B = \frac{1}{5}$ $0 = -9A + 5B + \frac{3}{2} - \frac{1}{10} \Rightarrow 9A - 5B = \frac{7}{5}$ $A = \frac{6}{35}, B = \frac{1}{35}$ $y = \frac{6}{35}e^{-9x} + \frac{1}{35}e^{5x} + \frac{3}{2}xe^{5x} - \frac{8}{225} - \frac{1}{10}x - \frac{3}{5}x^2$
	Obtains the correct Complementary Function.	1.1b	A1	
	Deduces correct exponential form of Particular Integral, Cxe^{5x}	2.2a	B1	
	Uses correct polynomial form of Particular Integral, $D + Ex + Fx^2$	3.1a	B1	
	Substitutes their y'_{PI} and y''_{PI}, with their y_{PI}, into the differential equation.	1.1a	M1	
	Compares coefficients to obtain at least three of their C, D, E and F (with at least two correct).	1.1a	M1	

	Obtains correct values of C, D, E and F	1.1b	A1	
	Forms their General Solution, with exactly two arbitrary constants.	3.1a	M1	
	Uses their $y_{GS} = \frac{37}{225}$ and $y'_{GS} = 0$ to form two equations in their two arbitrary constants.	1.1a	M1	
	Obtains the correct final result.	3.2a	A1	
	Question total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
20(a)	Uses integration by parts.	3.1a	M1	$I_n = \int_0^{\frac{\pi}{4}} \cos^n x\,dx = \int_0^{\frac{\pi}{4}} \cos^{n-1} x \cos x\,dx$ $u = \cos^{n-1} x \qquad v' = \cos x$ $u' = -(n-1)\cos^{n-2} x \sin x \qquad v = \sin x$ $I_n = \left[\sin x \cos^{n-1} x\right]_0^{\frac{\pi}{4}} + (n-1)\int_0^{\frac{\pi}{4}} \cos^{n-2} x \sin^2 x\,dx$ $I_n = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^{n-1} + (n-1)\int_0^{\frac{\pi}{4}} \cos^{n-2} x\left(1-\cos^2 x\right)dx$ $I_n = \left(\frac{1}{\sqrt{2}}\right)^n + (n-1)\int_0^{\frac{\pi}{4}} \cos^{n-2} x\,dx - (n-1)\int_0^{\frac{\pi}{4}} \cos^n x\,dx$ $I_n = \frac{1}{2^{\frac{n}{2}}} + (n-1)I_{n-2} - (n-1)I_n$ $nI_n = \frac{1}{2^{\frac{n}{2}}} + (n-1)I_{n-2}$ $I_n = \left(\frac{n-1}{n}\right)I_{n-2} + \frac{1}{n\left(2^{\frac{n}{2}}\right)}$
	Obtains a correct result of integration by parts.	1.1b	A1	
	Substitutes limits into first expression on RHS and simplifies.	1.1a	M1	
	Uses a trig identity to obtain an equation involving I_n and I_{n-2}	3.1a	M1	
	Rearranges to make I_n the subject of a three term equation .	1.1a	M1	
	Completes a rigorous argument to reach the required result. AG	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Marks	Typical solution
20(b)	Obtains and uses $I_0 = \frac{\pi}{4}$	1.1b	B1	$I_0 = \frac{\pi}{4}$ $I_2 = \frac{1}{2}\left(\frac{\pi}{4}\right) + \frac{1}{2}(2^{-1})$ $= \frac{\pi}{8} + \frac{1}{4}$ $I_4 = \frac{3}{4}\left(\frac{\pi}{8} + \frac{1}{4}\right) + \frac{1}{4}(2^{-2})$ $= \frac{3\pi}{32} + \frac{3}{16} + \frac{1}{16}$ $= \frac{3\pi}{32} + \frac{1}{4}$ $I_6 = \frac{5}{6}\left(\frac{3\pi}{32} + \frac{1}{4}\right) + \frac{1}{6}(2^{-3})$ $= \frac{15\pi}{192} + \frac{5}{24} + \frac{1}{48}$ $= \frac{15\pi + 44}{192}$
	Uses the formula three times.	1.1a	M1	
	Completes fully correct working to obtain $\frac{15\pi + 44}{192}$	2.1	R1	
	Subtotal		**3**	

	Question total		**9**	

	Question paper total		**100**	